INTRO TO HIGHER MATH
SUGGESTED PROBLEMS – DO NOT TURN IN

(1) Let $L, K \in \mathcal{D}$, where $\mathcal{D}$ denotes the set of Dedekind cuts. Show that
$$L + K = \{a + b \mid a \in L \text{ and } b \in K\}$$
is a Dedekind cut and satisfies
$$i(a + b) = i(a) + i(b),$$
where $i : \mathbb{Q} \to \mathcal{D}$ is the injection defined by $i(a) = \{x \in \mathbb{Q} \mid x < a\}$.

(2) Let $L \in \mathcal{D}$.
  (a) Prove that $L \cdot i(1) = L$.
  (b) Prove that $L + (-L) = i(0)$.

(3) Let $X \subset \mathcal{D}$ be bounded below. Prove that X has a greatest lower bound.