

**INTRO TO ALGEBRAIC TOPOLOGY**  
**HOMEWORK 8 DUE APRIL 9**

Turn in the following:

- (1) Hatcher Exercise 2.1.11 (p. 132)
- (2) Hatcher Exercise 2.1.12 (p. 132)
- (3) Hatcher Exercise 2.1.14 (p. 132)
- (4) Hatcher Exercise 2.1.15 (p. 132)
- (5) Let  $A$  and  $B$  be chain complexes. A chain map  $f : A \rightarrow B$  is a *chain homotopy equivalence* if there exists a chain map  $g : B \rightarrow A$  such that  $f \circ g$  and  $\text{id}_B$  are chain homotopic, and  $g \circ f$  and  $\text{id}_A$  are chain homotopic.
  - (a) Prove that if  $f : A \rightarrow B$  is a chain homotopy equivalence, then  $f$  induces an isomorphism on homology.
  - (b) Give an example of chain complexes  $A$  and  $B$  with isomorphic homology but no chain homotopy equivalence between them. (Hint: Let  $A$  be  $\mathbb{Z}$  in two consecutive gradings and zero everywhere else.)

Think about the following (but do NOT turn in):

- Let  $0 \rightarrow A \rightarrow B \rightarrow C$  be a short exact sequence of chain complexes. Finish the proof from class that this induces a long exact sequence on homology.